An example of incorrect behaviour of a senior mathematician

towards a young mathematician

by Laurent Lafforgue (July 2016)

On page xvii of the book manuscript "Periods and Nori Motives" (http://home.mathematik.uni-freiburg.de/arithgeom/preprints/buch/buch-v2.pdf, version 2 of May 13, 2016) one reads the following sentence :

"*The category theory aspect of the construction of Nori motives has been generalized. (…) Barbieri-Viale, partly together with his coauthors Carmello, Lafforgue and Prest, have taken the generalization much further, see [BCL], [Bar], [BP].*"

Actually, as I explain in this document, the main contributor to the generalization of Nori motives is Olivia Caramello, not Luca Barbieri-Viale. Nonetheless, for people who don't know the full story, which I tell below, he appears as the main contributor because he signed three papers whereas in reality :

- he didn't contribute to the main paper [BCL],
- the construction of "T-motives" in [Bar] is just a trivial variant of Caramello's construction,
- he succeeded in signing [BP] whose main result had been found by Prest before.

This unfortunate outcome is the result of a series of actions taken by Barbieri-Viale which display a strategy to attach his name to a new construction in place of its true author. In fact, as argued below, the slides of his talk in Rutgers, his papers [Bar] and [BT] and what he wrote on his web page showed a constant tendency to pay to Caramello as little credit as possible, or even no credit at all, and to grab for himself as much credit as possible.

I write these notes as a "co-author" of the first paper [BCL] and as a witness of all Caramello's work on Nori motives and related subjects.

I. The paper [BCL] "Syntactic categories for Nori motives" (http://arxiv.org/abs/1506.06113) signed by L. Barbieri-Viale, O. Caramello and L. Lafforgue

Already in the first version (June 19, 2015) it was written at the end of the introduction :

"*As it is clear enough from the language and methods used in this paper, the syntactic construction and the main theorem are due to the second-named author.*"

In the second completed version (May 15 , 2016), it is more precisely written :

"*The first named author and the third named author want to recognize here that the construction and the results of this paper are due to the second named author. They originated in a question raised by the first named author on the possibility of reinterpreting Nori's construction in terms of the theory of classifying toposes, which the third named author had talked to him about. The second named author benefited from many hours of conversations on algebraic geometry with the third named author and the first named author.*"

I will tell more later on the full story. In fact, I had already told part of the story in the introduction of a course (http://www.ihes.fr/~lafforgue/math/CoursMotifsNoriCaramello2015.pdf) I gave at IHES in September and October 2015.

The question of Barbieri-Viale of possibly reinterpreting Nori motives in terms of classifying toposes dates back to a message sent to me on April 12, 2013 (see section III). In the same message, he indicated he had begun to work on this question with a colleague in Milan who is a specialist of categorical logic and toposes, Silvio Ghilardi.

During the following year, they were not able to progress and didn't write anything. In March 2014, Ghilardi dropped out of the project and Barbieri-Viale, following my advice, asked Caramello to join.

She first spent months reading many papers in order to learn the basics of algebraic geometry, motives and the Nori construction. She even decided (and succeeded) to learn enough German to read von Wangenheim's memoir, “Nori-Motive und Tannaka-Theorie”.

At the end of 2014, she began to write the programmatic text “Motivic toposes” (http://arxiv.org/abs/1507.06271) which addresses much deeper questions than the question of reinterpreting Nori's construction, in particular the questions of “independence of l” of l-adic cohomology theories. Her completely original approach is based on the notion of “atomic two-valued toposes” which she had studied and developed extensively in several of her previous papers ever since her PhD thesis in 2009.

She would have liked a lot to collaborate and regularly sent to Barbieri-Viale the successive versions of her text, waiting for him to begin to contribute, but he never contributed even a line.

In May 2015, she decided to come back to the more elementary question of reinterpreting Nori's construction and, to my astonishment, did it in just a few days. I was a witness of that as she was at IHES and told me about her progress almost every day.

She announced that to Barbieri-Viale in a first message on May 26 (I was in copy), sending to him 7 pages of the text she had already written : it already contained the necessary preliminaries of regular logic, the statement and proof of the main theorem and the necessary and sufficient conditions for the universal abelian categories associated to different cohomology functors to be equivalent.

Barbieri-Viale just answered a few words, not even thanking her.

On June 6, she sent to him and myself a longer version of the text she had just written. It was already 16 pages long and contained everything of the later arXiv text except for :

- the abstract and the introduction,
- the review of Nori's original construction (paragraph 2.1 of the arXiv text),
- the annex paragraph on the relationship with comodules (paragraph 2.5 of the arXiv text).

Barbieri-Viale replied exactly three words, once again without thanking her.

He decided to visit us at IHES and arrived on June 15. I remember he first couldn't believe the result was correct. She had to spend a few hours to convince him.

After he was convinced came the question of who should sign the paper. The situation was delicate because this had been supposed to be a collaboration but there eventually had been no collaboration at all. Barbieri-Viale nevertheless signed the paper. My opinion was that Caramello should have signed the paper alone, for the obvious reason she was the only author : it would have been enough to indicate in the introduction that the origin of the paper was a question of Barbieri-Viale. Eventually she was too kind (or felt too weak as he is a well-recognized professor and she is a young mathematician still in a precarious position) : she accepted him to sign the paper and asked me to also sign it “because she had talked about these topics much more with me than with him and I had taught her some algebraic geometry”. I accepted on the condition that the introduction of the paper indicates to whom the results had to be credited.

Barbieri-Viale stayed at IHES for the June 15-19 week and the paper was completed :

- Caramello added a paragraph on the relationship with comodules,
- Barbieri-Viale provided some private notes of his which served as a basis for the paragraph recalling Nori's original construction (this is his only contribution : no research work there),
- I wrote the abstract and the introduction (this is my only contribution : no research work either).

The text was submitted on arXiv on June 19.

In messages I sent to him directly afterwards, I wrote repeatedly that neither he nor me deserved to have signed the paper [BCL]. I suggested we should try to deserve it a little by adding to the paper extra contributions which we would work out ourselves. I even suggested we could try to take care of some side questions which still had to be treated.

My own contribution eventually consisted in giving lectures at IHES in September and October about Caramello's work, first explaining in the introduction that everything was due to her. Paragraph I.3 of my lecture notes, which detailed the consequences for motives of her work, was eventually translated into English and integrated to the second version of [BCL] as chapter 3.

As to Barbieri-Viale, it is a fact that he didn't even try to add anything to [BCL]. It was enough that he had signed it.

II. Some mathematical explanations

Caramello's extremely general construction process consists in the following steps :

(1) Starting from a representation T : D → A of a quiver D in the category A of R-modules over some arbitrary ring R (or, more generally, in the category A of R-linear objects of some arbitrary regular effective category), she defines the regular theory of T.

The signature of this theory only depends on D and R : its sorts are indexed by the objects of D while its function symbols are associated to the arrows of D and to pairs consisting of an object of D and the name of a structure map expressing R-linearity, that is addition, zero constant map and multiplication by the scalars in R. The axioms of the theory consist in all regular sequents which are verified by the representation T.

(2) She considers the "syntactic regular category" of the regular theory under consideration and verifies it is additive and R-linear as the starting regular theory is additive and R-linear.

(3) She considers the "effectivization" C of this syntactic regular category and verifies it is an abelian R-linear category as the syntactic regular category is additive and R-linear.

(4) She verifies that the representation T : D → A canonically factorizes as the composite D → C → A of a representation D → C and a faithful exact functor C → A. The category C together with the representation D → C is then shown to be universal for this factorization property, which is Nori's universal characterization.

A key point of this verification is the fact that for any factorization of T through a faithful exact functor of abelian categories C' → A as D → C' → A , the faithfulness property of the functor C' → A can be interpreted as meaning that D → C' is a model in C' of the regular theory of T.

All these ideas, construction steps and verifications, which combine several basic constructions of categorical logic, were found by Caramello and are completely new in the subject of Nori motives.

They allow in particular to get rid of the finite dimension or finite type hypotheses in Nori's original construction.

Another important feature of this construction process is that it doesn't use the notion of classifying topos (contrary to Barbieri-Viale's initial question) but the notion of syntactic regular category, which is on the syntactic side of categorical logic and not on the semantic side as classifying toposes.

It also has to be remarked that steps (2), (3) and (4) apply just as well to an arbitrary R-linear additive regular theory (or even a non additive regular theory) and associates to such a regular theory an R-linear abelian category (or an effective regular category) characterized by a universal property which corresponds to the starting regular theory.

Furthermore, Caramello verifies that if we consider two representations T : D → A and T' : D → A of D, there exists an equivalence between the associated R-linear abelian categories C and C' which exchanges the representations D → C and D → C' if and only if the regular theories of T and T' are the same.

Here are the main consequences for motives of her construction and results :

(i) Any "Weil" cohomology functor with coefficients of characteristic 0 defines a Q-linear category of Nori motives. It works in particular for l-adic or p-adic cohomology over an arbitrary base field of arbitrary characteristic, not only for Betti cohomology over a subfield of C as in Nori's original construction.

(ii) The categories of Nori motives (or Nori-Caramello motives ?) associated to the different "Weil" cohomology functors are equivalent if and only if the so-called "regular theories" of these functors are the same.

(iii) This condition is also equivalent to the existence of a category of mixed motives in Grothendieck's sense (i.e. a Q-linear abelian category which factors all "Weil" cohomology functors through faithful exact functors). If these conditions are satisfied, motives can be considered as objects in logic, more precisely as "syntactic" objects for the "regular fragment of first-order logic" (whereas cohomology functors appear as "semantic" objects). In some precise sense, motives are the family of "regular" Q-linear properties that should be shared by all "Weil" cohomology functors.

III. Some unreasonable pretensions which suddenly appeared in June and July 2015

Whereas Barbieri-Viale had not written anything from his April 2013 message to me up to the moment when Caramello completed the paper [BCL], and even though he could still at last begin to collaborate, he started – after coming back to Milan from his IHES visit – to spend time and energy exchanging with me many e-mails. To my surprise, he was claiming credit for most of what Caramello had done alone and made me lose my time trying unsuccessfully to remind him he had not made any work.

Forgetting that he had failed to make any progress in the year following his April 2013 message, when he was supposed to work with Ghilardi, and that he had not contributed at all to Caramello's work in the following year, he was claiming that :

(A) the idea of using toposes in the context of motives and of defining a “motivic topos” which would classify cohomology theories was expressed for the first time in his April 2013 message, so that the whole subject was “his programme” and he could compare his message with the Weil conjectures (!),

(B) most ideas Caramello used for her new construction were already present in his message.

On point (A), it has to be remarked that his message didn't contain any definition, result or conjecture. Here is the full text of this message (which I reproduce because he bases all his pretensions on it and recently circulated it to a list of colleagues together with comments which were not sent to me) :

“*Dear Laurent,*

as puzzled by your enthusiasm on "classifying topos" I think that it is feasible to work out a presentation of motives in the sense that I draft here below.

Let D be an oriented graph and let T:D --> Ab be a diagram of (finitely generated) abelian groups. Here T has to be thought as "homology theory" Note that I'm just adopting Nori's point of view in constructing "effective homological motives" where D = a suitable graph obtained from schemes (over a field k embedded in the complex numbers) and T = singular homology of pairs. Alternately, one can consider the category Sch_k (of pairs of) k-schemes and a functor T : Sch_k -->Ab satisfying a set of axioms (e.g. Bloch-Ogus axioms). In both cases I think we should be able to translate this T in a geometric theory of first order.

Assume that a classifying topos E[T] for T-models exists such that T-Mod (E) = Hom (E, E[T]) for any topos E (over sets). That means we have a syntactic site on D (or the category associated to D) and an universal model providing the "motivic" site and the "motivic topos" for schemes. Now denote A[T] the abelian category Ab (E[T]) of internal abelian groups which is the abelian category of "mixed motives" for schemes. Any "model" i.e. morfism f: E --> E[T] induces an exact functor f^*: A[T] --> Ab (E) which is the "realisation" functor (which is also faithful if f is a surjection)

Now I think that (by general non-sense) we also get in this way a result of Nori that there is a factorization D --> A[T] --> Ab of the given diagram T which is universal among all such factorisations through abelian categories (over Ab).

How it sounds!?

All the best, L.

Ps. I started to check the logic items with Silvio Ghilardi a colleague here in Milano which knows well model theory & also is not afraid of topos theory! I hope we can provide a more detailed version soon!”

In order to understand the situation, one has to know that an extremely general theorem proved by logicians in the 70's allows to associate to any first-order (geometric) theory (in logic's sense) a Grothendieck topos, called its “classifying topos”, which incarnates the mathematical or semantic content of the theory. The classifying topos of a theory is defined by a site, called the “syntactic site” of the theory, consisting in a category, called the “syntactic category”, endowed with a Grothendieck topology, called the “syntactic topology”. The passage from the syntactic site to its associated classifying topos incarnates the passage from the logical presentation of a theory to its mathematical content, from syntax to semantics. The classifying topos of a theory verifies in particular the property that its points identify with the set-theoretic models of the theory. This theorem had been more or less neglected for several decades up to the moment when, already in her PhD thesis, Caramello began to build a new theory on it.

This theorem means that toposes can be associated to all mathematical situations were it is possible to define first-order (geometric) theories axiomatizing at least part of the properties of the objects under consideration, i.e. everywhere in mathematics. But the fact to realise that this theory can potentially be applied everywhere – for instance in the domain of motives – doesn't mean you have done something.

For instance, I had already asked the question of applying classifying toposes to motives as well as to the Langlands programme in February 2013 (see : http://www.ihes.fr/~lafforgue/math/ExposeParisVII.pdf) but it doesn't mean I had done anything.

Real work consists in defining classifying toposes which can capture at least part of the essence of the situation and in proving new results with them. As classifying toposes are associated to theories, the question of defining "good" classifying toposes reduces to the question of defining good first-order theories, that is good families of axioms. The answer (or answers) to that question depend on the problems you are interested in.

Caramello has proposed two completely different answers in her two texts "Motivic toposes" and "Syntactic categories for Nori motives".

The programmatic text "Motivic toposes" is oriented to "independence of l" problems. Most of it is written to progressively work out a possible answer to the question of defining a theory whose classifying topos would be adapted to these problems. The answer it proposes is very elaborate and rests on many previous papers of hers.

The other text is much more elementary and considers completely different theories : the so-called "regular theories" defined by deciding that their axioms are all regular linear properties verified by a cohomological functor under consideration. In fact, as already remarked, Caramello's construction of motives in this text eventually doesn't use the notion of classifying topos.

On point (B), it has to be said that none of the ideas (recalled in steps (1), (2), (3) and (4) of paragraph II) used by Caramello in her construction were present in Barbieri-Viale's message : the idea to define the regular theory of a representation of a quiver, the idea to consider the associated syntactic regular category (which is different from the syntactic category of a first-order geometric theory), the idea to consider its effectivization, the idea to interpret the faithfulness property of the factorizing functor as meaning the factorizing representation is a model of the regular theory of the starting representation.

Barbieri-Viale's message to me, made more clear than how it really is, just hinted that there should exist a first-order theory T such that its classifying topos E verifies the following :
(1) Weil cohomology theories are points of E (= set-theoretic models of T).
(2) The category of motives is the category of abelian objects of E.
(3) The realization functors are the restriction of the fiber functors of the points of E.
(4) The universal property characterizing Nori motives follows from the universal property characterizing the classifying topos E.

It can be commented that :
- Property (1) is automatic for any theory T whose axioms are verified by Weil cohomology theories. Even empty theories (without axioms) would work !
- Property (2) is impossible : it has to be replaced by something much more subtle, for instance the full subcategory on "super-coherent" objects of E if T is additive regular. This is because of the fact, which is part of Caramello's discoveries, that motives are syntactic objects, not semantic objects as classifying toposes.
- Property (3) is automatic for any axiomatization T (including empty theories), except for the fact that the realization functors are expected to be not only exact (which is automatic) but also faithful, which puts extremely strict conditions on the theory T. If T is chosen as Caramello does in her paper [BCL] (assuming it doesn't depend on the Weil cohomology functor under consideration), it is all right. It T is chosen as Barbieri-Viale does in his paper [Bar] (just taking the standard axioms used by Voevodsky), I see no convincing reason to believe faithfulness could hold.
- Property (4) is wrong as Caramello has shown that the universal property of Nori motives has to be interpreted in another way.
- Properties (1), (2), (3), (4) are purely formal and do not have by themselves any concrete consequences for Weil cohomology theories and their relationships, unlike the other properties considered by Caramello in her two papers [BCL] and "Motivic toposes".

IV. The paper [Bar] "T-motives" (http://arxiv.org/abs/1602.05053) by L. Barbieri-Viale

A first version of this paper was submitted on February 16, 2016 and a second revised version on April 4, 2016.

As can easily be checked, Barbieri-Viale's construction of so-called “T-motives” in this paper is a trivial variant of Caramello's construction in the previous paper.

Let's recall indeed that Caramello's extremely general construction process applies to any (linear or even non linear) regular theory : it consists in associating to any such regular theory the effectivization of its syntactic regular theory and verifying it is characterized by some universal property which corresponds to the starting regular theory. In her paper [BCL], this construction starts with the “regular theory” associated to a given cohomological functor (or, more generally, a representation of a quiver in the abelian category of R-modules or even in the category of R-linear objects of a regular effective category).

In his paper, Barbieri-Viale starts with the simpler and coarser regular theory defined by the usual standard properties of cohomological functors (linearity, exactness, naturalness plus homotopy invariance and Mayer-Vietoris). After that, he just copies Caramello's construction process.

The only other differences are cosmetic :

- He refers to a general theorem of Tierney according to which any effective regular additive category is abelian, whereas Caramello gives a 20-lines direct proof.

- He replaces the word “effectivization” (used by Caramello) by the expression “Barr-exact completion”. It is confusing as this categorical construction is not due do Barr. It also makes a little more difficult for a non specialized reader to realize that he is just copying Caramello's construction process.

His way of presenting her construction process is not simpler, just because this process doesn't depend on the more or less elaborate character of the regular theory it starts with.

It can also be remarked that, for defining his so-called “T-motives”, Barbieri-Viale completely forgets Nori motives and only retains Caramello's construction process which goes well beyond the case of Nori motives.

In the first version of his paper, Barbieri-Viale had not even mentionned Caramello's name in the introduction. Only at the end of the paper, he expressed gratitude to Caramello, his Milan colleague Ghilardi and myself for “many helpful discussions”. To put Caramello, Ghilardi and myself on the same footing is misleading as the construction process he is using belongs to her alone. I certainly had discussions with him but everything I told him about classifying toposes I had learnt from her. Lastly, he had worked with Ghilardi from April 2013 to March 2014, trying to answer his initial question about Nori motives, but they failed, Ghilardi dropped out of the project and he asked her to join.

After I threatened him to expose the full story on the web, he reluctantly added in the introduction of the second version :

“The use of the (Barr) exact completion of the regular syntactic category was an idea of O. Caramello, appearing in [BCL] in order to obtain Nori’s category of a representation of a diagram via the regular theory of a model.”

But it would have been more respectful of the truth to write that not only this idea but the whole construction process belongs to her.

Lastly, let's remark that he writes in the paper's abstract :

“Considering a (co)homology theory T on a base category C as a fragment of a first-order logical theory we here construct an abelian category A[T] which is universal with respect to models of T in abelian categories.”

It would have been more respectful of the truth to replace “we here construct” by “we copy Caramello's construction process of [BCL], replacing her starting regular theories by a coarser one, to construct...” In fact, the completely general character of her construction and its universality with respect to models in arbitrary effective regular (in particular abelian) categories had already been noted by her in Remarks 2.6(b)(a) of the first version of her paper [BCL].

V. The paper [BP] “T-motives and definable categories” (http://arxiv.org/abs/1604.00153) signed by L. Barbieri-Viale and M. Prest

A first version of this paper was submitted on April 1, 2016 and a second revised version on April 15, 2016. It presents a purely categorical version (without logic) of Caramello's construction.

The alternative method of the paper was deduced from her construction by applying the general set-up of relations between categorical logic and abelian categories theory which is Prest's main subject of study.

In fact, Prest had already written to her on November 27, 2015 a message with an attached document in which he outlined in an essentially complete way his variant of her construction (and which she forwarded to me). The message didn't mention Barbieri-Viale. In fact, they didn't know each other before meeting at a conference on toposes she organized at IHES for the November 23-27 week.

A reading of the paper confirms it contains essentially a unique result, which is purely categorical, was already exposed in Prest's message and is clearly his mathematics. The only ingredients which can be associated with Barbieri-Viale are the easy links to Nori's classical construction (which is not his work), to Caramello's new construction of generalised Nori motives (which is not his work either) and to so-called "T-motives" (the trivial variant he introduced in the previous paper). This means that, once again, he succeeded in signing a paper presenting a result due to somebody else.

The paper constructs exactly the same objects as Caramello had done and is not more general : on the opposite, her construction process is more flexible and can be applied in much more general situations – non abelian or even non linear – as it had been remarked by her already in the first version of [BCL] (see Remark 2.6(b) page 14). Even in the abelian case, her construction process is more general as it applies to any starting additive regular theory.

In particular, the universal abelian category Ab(D) of their main theorem on page 3 can be easily obtained by Caramello's construction process, starting with the regular theory consisting in the additivity axioms and nothing else. As recalled in paragraph II, when considering a representation T of a quiver D, the language of the appropriate "regular theory of T" is associated to D while the axioms are all regular "sequents" which are verified by T. If the construction is applied to the "almost empty" regular theory with the same language and only the additivity axioms, the outcome is the Freyd free abelian category on the path category associated to D, which is Ab(D). Making the other axioms enter the place corresponds to taking a Serre quotient. This is what Prest had to understand to translate Caramello's construction into his own language.

Her construction is yet another illustration of her general point of view (which she explained in several papers and lectures) that the categorical logic passage from theories to their universal categorical models (in different possible senses) can be interpreted as an extremely general way to define mathematical objects by generators and relations : basic theories define free objects and extra axioms correspond to relations which define quotient objects of the free objects.

The statements of [BCL] are phrased for representations of quivers in categories of R-modules (just because cohomology functors usually take values in R-modules) whereas the statements of [BP] are phrased for representations of quivers in arbitrary abelian varieties. But it had already been observed in Remark 2.6(a) of the first version of the paper [BCL] that the construction works (word for word) for any representation of a quiver in the category of R-linear objects of any effective regular category (in particular of any abelian category). Unfortunately, the paper [BP] doesn't refer to this remark, either because Barbieri-Viale didn't read well enough the paper [BCL] he signed, or because he prefers to make readers believe that the paper [BP] is more general.

In the first version of this text, Caramello's name didn't appear anywhere, except in the bibliography as one of the three « authors » of the text [BCL].

I wrote to Prest who rightfully added in the introduction of the second revised version of this text :

"In [BCL], Caramello gives a proof of Nori's theorem by a very different, more general and rather direct construction..."

The first version of [BP] presented as a "fundamental result" the main theorem despite it is only a variation of Olivia Caramello's construction process.

Prest wrote to me on this point : "Perhaps also we could have chosen a better term than "fundamental result" but we wished to point out the route to this category starting from Freyd's old construction – which is, indeed, set within the additive context, so not as general as Olivia's."

The expression "fundamental result" has disappeared from the second version, but their main theorem is still introduced by the sentence "all the previously mentioned constructions can be deduced from the following result" which is not satisfactory as it implies Caramello's result is a particular case of theirs whereas theirs is just an alternative method to obtain it and their construction process is less general.

Lastly, one wonders why the title refers to “T-motives” and not to Nori motives (or Nori-Caramello motives ?).

VI. The slides (http://users.unimi.it/barbieri/topos.pdf) of Barbieri-Viale's talk at Rutgers on August 21, 2015

This was two months after Caramello had written [BCL]. The title of the talk is : “Nori motives and the motivic topos”.

The part pertaining to our subject is part III, which begins on page 14.

It is introduced by the vague formula “thanks to O. Caramello and L. Lafforgue” which is misleading because [BCL] is due to Caramello alone and he had learnt much more from her than from me. In fact, everything I had told him in relation with classifying toposes I had learnt myself from Caramello, and most of what he writes in this part was taught to him by her, either directly or indirectly through me.

On page 21, the main theorem is attributed to Caramello, myself and himself whereas it is entirely due to her.

VII. Barbieri-Viale's comments on his home page (http://users.unimi.it/barbieri/SP.html)

On this page, Barbieri-Viale talks in particular about “the motivic topos and Nori motives”. At the bottom of the page, he writes as footnote [5] :

“*I can tell that on 11 April 2013 exactly I had the idea of a "motivic topos". I remember it because I was traveling on a train from Brussels to Ghent after an early morning flight from Milano going to the EUA General Assembly 2013 hosted by Ghent University, Belgium, on 11 April 2013 as an official representative of the Rector of Milan University. I was a Rector's Delegate and I was also very busy with administrative duties those days. However, the picture of a "motivic topos" was so clear for me that I drafted in an email to L. Lafforgue the day after I got it.*”

Let's notice Caramello's name is not even mentioned there and, in the other comments on the same page, she is only mentioned as a co-organizer of a seminar in Milano in March 2014.

Another strange point is that, in the message he indeed had sent to me on April 12, he had written :

“*I started to check the logic items with Silvio Ghilardi a colleague here in Milano which knows well model theory & also is not afraid of topos theory! I hope we can provide a more detailed version soon!*”

One may wonder how it is possible that he had started to “check the logic items with Silvio Ghilardi” on April 12, for an idea he had on April 11 while travelling in Belgium.

He also fails to say his idea was “so clear” that Ghilardi and himself were not able to make any progress in the following year – so that Ghilardi eventually dropped out of the project, which he thought was impossible (if I remember correctly what he said in front of me in March 2014) – and that he had to ask Caramello to enter the project and to wait another year for her to solve alone the initial question of reinterpreting Nori motives (in fact not in terms of classifying toposes as he had asked, but of purely syntactic constructions of categorical logic).

Lastly, he fails to say how it happened that he learnt about classifying toposes and got interested into them :

On the occasions of a visit he made at IHES in June 2012 and of lectures I gave in Milan in December 2012, I had told him about the extremely general and important notion of classifying topos, which I had learnt from Caramello. This notion has been known since the 70's but more or less abandoned up to the moment when she gave new life to the subject in her PhD thesis and her subsequent work. I conveyed to Barbieri-Viale the enthusiasm for classifying toposes that had been given to me by her.

His message to me on April 12, 2013 was a follow-up of what I had told him in general about classifying toposes and their potentially almost universal use in mathematics. In fact, I was very happy he had listened to me (unlike many other algebraic geometers) and I am still grateful to him for that.

His message said that, “puzzled by my enthusiasm about classifying toposes”, he had thought about a possible application of this notion in the context of motives. In particular, he asked whether Nori motives could be recovered in this way (as both Nori motives and classifying toposes are characterized by universal properties).

This is the way Caramello later heard about Nori motives and began to work on them. Without the information on the universal property of Nori motives and the incentive to enter the subject, she would not have begun to work on motives and written her paper [BCL] nor the much more advanced programmatic text “Motivic toposes” which, I think, is the beginning of something important. Of course, the papers [Bar] and [BP], which follow [BCL], would not exist either.

Still, the importance of Barbieri-Viale's initial question should not be exaggerated : it was just a working hypothesis which, even in the very vague form in which it was stated, eventually proved false. It can even be argued that this working hypothesis was misleading since it proved incorrect.

In fact, her categories generalizing Nori's category are not the categories of abelian objects of classifying toposes (as he had supposed in his message) but rather can be realized (though this is not at all a natural way to define them) as the full subcategories of such toposes on the “super-coherent” objects, a much subtler notion.

As explained above, she constructed her categories much more naturally as the effectivizations of the “syntactic regular categories” of the “regular theories” of the (co)homogical functors under consideration (or, more generally, of representations of quivers). This means these categories are of syntactic nature, not of semantic nature as classifying toposes. The distinction between syntax and semantics is the most important distinction of logics.

The discovery that motives over an arbitrary base field are syntactic objects (if they actually exist) is completely new and is one of the main corollaries of her construction and results.

VIII. Another incorrect behaviour possibly prompted by Barbieri-Viale's example

On April 13, 2016, two weeks after the first version paper [BP] was posted on arXiv, Pierre Cartier made the following announcement of a talk at IHES scheduled for May 18 :

Title : “On a new construction of Nori motives”
Abstract : “I shall report about a new direct construction of Nori motives, discovered independently by Barbieri-Viale and Prest on the one hand, by Joyal and myself on the other hand. Unlike previous constructions, one uses only standard constructions in category theory, like Frey free abelian category on a given additive category, and Serre’s construction of quotient categories.”
(see : https://indico.math.cnrs.fr/event/1262/)

This announcement was incorrect for two reasons :

Firstly, through this announcement, Cartier and Joyal claimed equal credit for some work published by other people whereas they didn't publish anything and are not even able to prove that they had independently done the same work.

Secondly, Cartier called a “discovery” the alternate purely categorical method worked out by Prest for Caramello's construction. It is different enough to be considered actual work but it is just an alternate (less general) version of the same construction. So it is incorrect to call that a discovery, not to mention it is a variant of Caramello's work and not even to quote her name.

Cartier's announcement prompted shocked reactions by several people who, as well as him, had attended my lectures six months earlier. Eventually, his talk was completely different. He began the talk with some kind of historical introduction on the subject of Nori motives which paid full credit to Caramello for her work and recognized its significance. He even didn't try to grab credit for the variant of Caramello's construction process worked out in the paper by Prest.

Still, his announcement remains available on internet.

This episode shows that not only Barbieri-Viale but at least two other well-recognized professors have been driven to try to unjustly grab credit for Caramello's work on Nori motives.

It can certainly be interpreted as an indication of its importance.

IX. Conclusion

Barbieri-Viale had the opportunity to collaborate with Caramello who is an extremely original and creative young mathematician. Even after her work on Nori motives, it was still possible : in June and July 2015, I wrote to him repeatedly that I knew Caramello would be extremely happy to collaborate, first on the Nori motives paper which still could be completed with extra parts and later on other papers on the motivic subject if only he contributed substantially to them.

But he wasted this constructive opportunity to try to grab credit from her for a result which, for her, is just the beginning of her research on motives.

Eventually, his grabbing attempt is going to fail because I am defending the truth, he has lost the opportunity to collaborate with her, his reputation is tarnished, he has destroyed the friendly relationship we had, he has obtained that I shall never again visit him at his university.

What a waste and what a pity !

I am all the more saddened by this outcome as his April 2013 message had made me very happy – because it showed he had listened (unlike other algebraic geometers) to what I had told him about the theory of classifying toposes – and as his initial question eventually gave Caramello the opportunity and incentive to enter the subject of motives and to already make a very interesting original contribution.

This is unfortunately not the first time well-established and well-recognized professors tried to grab credit from Caramello for some of her works or to outright steal results from her. I wrote about the particular case of Barbieri-Viale – which is actually not even the worst she faced – because I was a direct witness of Caramello's work on motives and of his behaviour and because I can prove everything I write on this matter.

It is clear that if such things happened to Caramello, they can also happen, and certainly actually happen, to many other young mathematicians or scientists.

This means there exists a tendency among well-established professors to consider that they have more or less all rights on young researchers still in precarious position and that ethical rules apply only between colleagues of comparable status. I still hope and think that this tendency manifests itself only in a minority of professors, but it can be observed that even the professors who are completely correct with young people still in precarious position are reluctant to give up solidarity with their colleagues when they behave incorrectly.

It also highlights one very bad effect of precariousness for young scientists who, for this reason, are in a weak position and feel high pressure from their more fortunate elders. This is not good, not only from a human viewpoint, but also from a scientific viewpoint as it means that young scientists – who, in principle, should bring novelty – find it hard to develop their work independently of well-established professors and to get appropriate recognition for their truly original work. The example of Caramello shows that if a young mathematician still in precarious position develops a new personal point of view in mathematics and a new theory, then he is exposed to attacks by well-established professors.

This raises two general most important questions for the mathematical, scientific or academic community :

Firstly, what to do when some well-established colleagues behave in an incorrect way towards young researchers who are not in a position to defend themselves ?

Secondly, shouldn't the contemporary development of precariousness for young scientists be questioned ?