

Speculation and narration in mathematics*

by Laurent Lafforgue¹

The inspiration for the topic of this conference came to me from two pages by the young French philosopher Fabrice Hadjadj, published in a recent book of interviews “L’héritage et la promesse² (“Legacy and promise”)”.

Asked about the key question on the articulation between theology and sciences, Fabrice Hadjadj replies that “if science seems to us so far removed from faith, it is not so much because of the supposedly insurmountable obstacle between experiment and faith, as the fact that 'truth', as most often represented by sciences, blanks out the proper noun, rejects actual existence, ignores the relationship.” “Truth tends to be thought of as seeing, and not living. Knowing truth is understood as being in the position of a spectator dominating his object. In that sense, contemporary technology, television, the virtual world, would all appear to emanate from this concept of truth, where seeing takes over from living. You could say that this is a concept of truth that is not nuptial but pornographic: we want to see love made, possibly make love, but not live it. For truth to be nuptial, it needs to resemble an embrace: I must give up being an invulnerable spectator to enter into a relationship with someone.”

Christ’s extraordinary words to Thomas “I am the way, the truth and the life” (John 14,6) do indeed reveal the fact that “truth will not be found in general terms, in what could be called anonymous clauses; no, Truth is a person, with his uniqueness, with a proper Name of which Peter says: ‘there

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²His Grace Dominique Rey and Fabrice Hadjadj, interviewed by Serge Sarkissian, On sime  ditions, April 2011

is no other name under heaven given to mankind by which we must be saved.’ (Acts, 4, 12)”

Which is why, says Fabrice Hadjadj, “in theology, exploring truth always requires both a speculative and a narrative approach, it has elements of both science and biography. Yet, for us, biography and science have been separated, speculation and narration have been perceived as opposites.”

The intuition we have of the existence of truth and of its unity therefore prompts us to think of sciences from the point of view offered by Fabrice Hadjadj: are they purely speculative and in accordance with the way we currently represent them, or are they also narrative, and therefore closer than is usually acknowledged, to the model of the Revelation? Are they purely theoretical - that is, etymologically, visual - or are they also life experiences? Are they completely impersonal or do they also include at least in part relationships with people?

It seems perfectly natural to consider mathematics, as it is in the first analysis seemingly the most speculative among the sciences, the most theoretical and the most impersonal of all, as well as one that provides the models for modern physics and, to a lesser extent, to other sciences. The mathematician that I am therefore believes it quite justifiable for me to speak in front of philosophers and representatives from other fields of learning and - because that is the crux of the matter - address the link between this unique science and truth.

Let us first ask ourselves what thoughts would go through the mind of a person, who was a complete stranger to mathematics and who, for the first time in his life, were invited to visit a university department or research centre dedicated to this discipline.

The first thought to take hold of his mind after his visit would surely be, that, in a mathematics centre, there is “nothing to see”: in other words, there is no spectacular experimentation equipment, no out of the ordinary object that might spark some curiosity or interest, just corridors, offices, rooms and amphitheatres furnished only with chairs, tables and seats, chalkboards or sometimes wipe boards, the sort of computers and keyboards that you see everywhere nowadays, books stacked in shelves, a few printers and photocopiers, some printed, some blank paper and pens. Thinking about it a little more, the visitor might also reflect, that, as well as

being very common and uninteresting, the objects seen are few. In comparison with other places of social activity - such as shops, transport, industry, construction, craft, hospitals, police, army, places of mathematics would seem to comprise an unusually far higher proportion of people to material objects. Only the teaching places - schools and high schools - and those other places of abstract and speculative activity that are banks and insurance companies seem to offer a similar appearance: everyday objects, few of them in fact and proportionally many faces.

It is not just the case that places of mathematics *look like* teaching places: they *are*, mostly. Our visitor could attend lectures, and if he did, would first and foremost be struck by the fact that he understood nothing, that all the sentences were woven with words the meaning of which escapes him, or more intriguingly still, words that are familiar in the common language, but that become incomprehensible in the context of the sentences spoken or written on the board. He could also attend seminars, that is, sessions similar to lectures, but where, as he could see, mathematicians would be teaching other mathematicians, and not students, another cause for surprise and questions.

If, for want of anything interesting to look at, the visitor tried to catch the people who inhabit places of mathematics in all their activities, he could also overhear informal conversations among mathematicians - and find then, not perhaps without feeling a degree of terror, that it seems natural for these people to speak in a casual way with words no less obscure than those used in lectures and seminar presentations. He could also observe many people in offices busy reading - reading books as well as printed texts or texts bound into what these people quaintly call journal fascicules (in French, the same word is used for a journal as for a newspaper) - or busy writing on a keyboard or with a pen. In the last instance, he could catch a few people apparently lost in thoughts more mysterious than everything else.

In short, the visitor would conclude that places of mathematics are inhabited mainly by people who are busy talking and listening in large and small groups, or reading, writing and thinking alone.

So, are mathematicians related to writers, journalists, orators or activists, who commit to the cause presented in the talks they hear, or else to friends putting the world to rights, chatting over a café counter?

To attempt a reply to these questions, there is no other way than to explore the contents of mathematicians' lectures, presentations,

conversations, journals and books. What are they in fact talking about in their prolific speech and writing, what are they expressing to one another, and what are they expressing to themselves when they think?

For this reason, our visitor would need to venture into opening articles and books written and read by mathematicians.

Even if not one word made sense, he would be struck by the extremely structured and organised appearance the texts display: they are broken down into parts, chapters, paragraphs and subparagraphs, each with a title or at the very least a sequencing and identification number. In addition, they comprise clearly identified and themselves numbered statements, with names such as “definition”, “lemma”, “proposition”, “theorem”, together with well framed expositions introduced with words like “notation”, “proof”, “remark” etc. The visitor would also notice that these texts are riddled with internal and external references and that these references obey a simple rule: they refer practically always to parts of the text situated above, that is read beforehand, and to articles or books published previously.

Our mathematics visitor would therefore be led to observe that each mathematical text is like a step on a journey, or a path, as if it told a story in chronological order and, what is more, as if the specific story it appears to tell were part of a general step in mathematical science and in the history of that onwards march.

However, the visitor would necessarily make a comment that seems to contradict the previous one: whereas any text is structured like a story and is explicitly linked to the wider story that mathematics as a whole appears to form, it is written in the present tense. As if it were a story without events, or rather a story telling events outside or beyond time.

Our visitor would then understand that mathematical texts belong to the writing and reading timeframe, as well as to the publications timeframe, but that the temporal structure of their neatly ordered story telling mirrors the intemporal structure of what they are telling the story about. The chain of logical implications that weave the fabric of deductive reasoning is displayed in the layers of linear reading over time and carefully dated publications.

Through the linear progression of single stories they each expound, and in the vast story they all make up together, mathematical texts reveal logical implications as if they were cause and effects linked over time, as if

logic were a form of the principle of causality, as if logical and temporal structures identified one another. But, the use of the present tense in the texts' verbs mean that, in its very essence, mathematical logic does not depend on time.

And yet, our visitor could wonder, any mathematical text is well and truly anchored in time: it not only includes a logical structure in the timeframe of a story, it is itself included in the timeframe of mankind as a moment and stage in the long history of the science it belongs to. Most concepts, results and methods that are part of it or that make it possible are borrowed from other articles and books, or from other mathematicians, long gone or the author's contemporaries. Any mathematical text is both a story, that is a temporal form, of an intemporal mathematical story, and the hinge - between past and present - of the history of mathematical science in its own timeframe. This duality in the identity of mathematical texts would in fact be particularly manifest in our visitor's eyes in the fact that very many mathematicians' names are used to designate mathematical objects and results, which is another way of linking the intemporal substance of mathematics to their history made by mankind.

Our observant visitor could also discover that mathematical texts veer between, on one hand, the use of prior methods, concepts and results in new analyses in order to solve problems set since a specific time and, on the other hand, the development of new methods, the introduction of new concepts, the exploration of new territories, the formulation of new statements, the demonstration of which follows, or are suggested as conjectures. Each of these two models could then appear to be both narrative and speculative, but in reverse: the model using established theories in new analyses to prove well known conjectures fits in more easily in the history of mathematics, in the illustration, summary and, in a way, in the renewed telling of one of its pages. It is however more speculative from the point of view of the substance of mathematics, as it consists in solving given problems by working back in the chain of their causes, to find the implications from which they derive. On the contrary, the model of discovery and development of new theories is more speculative from the point of view of mathematical history, as it equates to going back to the source of ancient theories so that, from those sources, another path is followed, but is more narrative from the point of view of mathematics, as it flows down the causal chain, just as water flows down a river bed.

Mathematical texts that expound or introduce a theory are indeed similar to narratives, more specifically to travel tales. The entirely discursive nature of speech or writing prevent an entire theory from being presented in one scene: its paths need to be travelled slowly, its crossroads negotiated, its towns, lands and buildings need to be visited one by one.

Our still uninitiated visitor could finally ask himself whether mathematical texts comprise events in the stories they unfold. He would conclude that, from the history of mathematics angle, this is certainly the case : the solution to an old problem or the emergence of a new concept represents historical events. But, from the substance of mathematics angle, do events exist? If it is true that mathematics are outside of time, can anything happen there? Our visitor would doubtless avoid answering such a strange question too quickly. He would simply note that, when mathematicians talk through this or that theory or demonstration, they often use such expressions as “Here is where something happens.” As if striking statements, an unexpected simplification, a computation result, gave these mathematicians the feeling they were moving from one landscape to another, opening a new vista.

But if it is true that mathematicians’ texts and speeches, each individually and all together, have the form of narratives, our visitor might well ask for whom are mathematicians telling their stories?

However paradoxical it may seem, given the manifestly unentertaining nature of these texts and the difficulty in understanding them, an objective fact would immediately spring to our visitor’s mind: lectures are given so that students can follow them, seminar presentations are made for colleagues to listen to them, conversations are held so that mathematicians can enter into a dialogue with one another, articles are written to be published, and finally, journals and specialised books are commercialised so that they can be bought – especially by university libraries – and read by mathematicians.

Our visitor would then ask what inclines mathematicians to teach students, give presentations for the attention of their colleagues or speak to them informally, to write articles or books intended for publication. He would then learn that what those institutions which employ mathematicians expect from them is that they give mathematics lectures and presentations, and publish articles or books, in exchange for the means of subsistence provided to them. This information would, however, doubtless only

displace the question in his mind, and make him address those formative years, when longing to become a mathematician appears and takes shape: why would young people ever engage in careers, of which the objective reality for them will consist of speaking to be listened to and writing to be read? It is obvious that this fact echoes far deeper human aspirations that dwell within them. So what can these aspirations be?

Our visitor could first recognise in most researchers in mathematics – as in all academic fields – the avatar of a desire, highly visible in children, and probably still present in adults, although in a more concealed manner: the desire to obtain confirmation from others, to draw their attention, and to receive their approval of what they do, which is felt in part to be approval of who they are. In other words, the desire to be loved, more or less corrupted and confused by the tenacious illusion that it is possible to conquer love by merit. Young children seek their parents' approval of what they do every day; to grow, they need to feel their parents' benevolent and supportive gaze on them. Later and in parallel, the desire to receive positive judgments from their teachers and professors represents one of students' most powerful reasons for working diligently. Our visitor, discovering the world of mathematics, would not fail to liken this well-known characteristic of the human heart to the importance afforded to small and great honours in university life and the prestige attached to certain places, names, institutions and to certain journals, etc.

Acknowledging this could lead our visitor to interpreting the obligation, which mathematicians and other academic researchers have placed upon themselves, to try to be listened to and read, as a sign of immaturity.

Unless he realised that another very deep human longing dwelt in researchers side by side with the previous one, and in the purest hearts, dominated it: the longing to share. In other words, the need to offer others a form of love, by sharing with them objectively precious things, which we possess or hope to be able to discover and make known. People know deep in their hearts that giving brings more happiness than receiving. For mathematicians, there is no other possible gift than speech, and therefore, our visitor would think, no true happiness is possible unless the narrative content of mathematics is liable to be of considerable value. Mathematicians necessarily compose their articles and books - that is, the tales of their mathematical journeys - thinking that the regions these tales describe are beautiful and worthy of being known and visited by others.

They are propelled by their desire to make known and loved their mathematics pathways and landscapes, the wealth and splendour of which they discover; in the same way are we happy to invite friends to a beloved country and try to share with them all the knowledge that this country gives us.

But our visitor would soon observe that the efforts deployed by mathematicians to share with others the truth and beauty they discover encounter similar obstacles and disappointments to those we suffer from when we invite dear friends to a familiar region, where everything speaks to our hearts, but not to theirs. Nowadays especially, mathematicians seem to be moving away from one another at great speed and, whereas the number of published articles and books never ceases to increase, that of actual readers of almost all these articles and books is undoubtedly constantly decreasing.

Mathematicians expound their narratives in the hope of sharing, that is, of experiencing a form of love with their human brethren through mathematics; but most will know the secret misery of the fruit of their mind's labour awakening no particular interest, as if they were talking and writing in a desert without ears or eyes. Even the most highly regarded mathematicians, those whose presentations and writings are awaited, listened to and read, generating other work, are not sheltered from self-doubt and despair; this can arise if they become conscious that, in the tokens of respect and admiration they are surrounded by, fascination for social glory and strength, or even jealousy, dominate and supplant the shared love for truth.

Our visitor could also ask mathematicians the difficult question of whether they prefer to read or write mathematics. Judging by the ever increasing flow of articles sent to the editorial boards of journals and by the matching increasing difficulty in finding reviewers for the articles submitted, our visitor would probably hear most mathematicians reply that they prefer to write mathematics. And this is humanly quite normal, the visitor would reflect, as there is more joy in giving than in receiving.

This clearly identified fact would then bring the visitor to ask himself whether writing a mathematical narrative intended to be read is indeed a form of gift to other mathematicians, if the latter do not in fact appear to be very interested in reading them. And if writing mathematical narratives represents a gift of questionable value to others, then why write?

Perhaps to write to one's self?

Our visitor would observe in any case that, as any text, mathematical texts are prepared and drafted by their author before being read by others: at least chronologically, writing precedes reading, with the result that a mathematician who writes has no other witness than himself. This general nature of writing is particularly strongly marked in mathematics, because, when compared to other sciences, time flows in it slowly: the time taken to craft books and articles is measured in months, even years and articles are generally published long after they have been submitted to a specialised journal. Today, most articles are made available beforehand on the web, they are easier to download, but time is still needed to read them and most are probably never read.

If he were to ask mathematicians, our visitor would in fact learn that, when they write up their work, mathematicians are not thinking about their future readers or anybody. They even forget themselves. They think of nothing but the object of the narrative, from which they are trying, through their writing, to reveal in their mind the clearest possible picture. A mathematical narrative is like a travel story in which, in fact, the journey consists of the narrative itself, the attentive traveller is its author and the pen, pencil or keyboard are both instruments of the tale and of the journey featured in the story. The paper or screen, a proxy for paper, is an ocean and the pen or keyboard, the stereotyped form of a pen, is a boat on that ocean.

But if it is true that mathematicians first *themselves* tell their tales *for* themselves, our visitor could ask: what is the point of this?

The question deserves all the more to be asked that mathematical writing requires considerable efforts of concentration and attention, at the very limits of the possibilities offered by the human mind. Our visitor might then reflect that telling one's self a story is only of true value if it is apparent that the story is not one's own, in other words that the story is inspired. The term inspiration is often used by widely differing mathematicians.

But most go no further. They talk of inspiration, without the idea entering their mind that it could come from an unknown person or author, from God, whose marvels every mathematician would praise and celebrate in writing and in speech.

Some mathematicians even say in the same breath that, to write mathematics, hearing needs to be finely tuned to the subtle, and very

discreet, almost inaudible voice of mathematical truths that are waiting to be told, to receive form in language; most, however, do not think that someone is speaking and yet, in laying down on paper their mathematical narratives, they show that they are paying attention to a certain kind of words of this someone and that they find their only joy in becoming servants to these words, following them as faithfully as they can.